

# School Systems and Efficiency and Equity of Education

Jung Hur

Changhui Kang\*

Department of Economics  
National University of Singapore

## Abstract

How students should be allocated to schools to achieve educational goals is one of important debates on the construction of public school systems. Promoters of comprehensive and selective school systems fail to reach a consensus on implications of each system for efficiency and equity of education. This paper examines impacts of different systems of student allocation on educational goals, using a simple economic model. It argues that how a selective system is designed matters a great deal in a comparison between comprehensive and selective systems: different designs of a selective system can yield widely different educational implications compared with those from a comprehensive system. A judicious use of a selective system can at times achieve educational goals better than a comprehensive system. Given our finding that different households prefer different school systems, we suggest that by offering multiple subsystems, the educational planner can enhance educational attainments of households beyond those achieved by a single national system.

*Key Words:* Education, Comprehensive and Selective School Systems

*JEL Classification:* D11, I20

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\* Corresponding author: Changhui Kang, Address: Department of Economics, Faculty of Arts and Social Sciences, National University of Singapore, AS2 1 Arts Link, Singapore 117570, Singapore, Email: [ecskch@nus.edu.sg](mailto:ecskch@nus.edu.sg), Tel: +65-6516-6830, Fax: +65-6775-2646.

# School Systems and Efficiency and Equity of Education

How students should be allocated to schools to achieve educational goals is one of important debates on the construction of public school systems. Promoters of comprehensive and selective school systems fail to reach a consensus on implications of each system for efficiency and equity of education. This paper examines impacts of different systems of student allocation on educational goals, using a simple economic model. It argues that how a selective system is designed matters a great deal in a comparison between comprehensive and selective systems: different designs of a selective system can yield widely different educational implications compared with those from a comprehensive system. A judicious use of a selective system can at times achieve educational goals better than a comprehensive system. Given our finding that different households prefer different school systems, we suggest that by offering multiple subsystems, the educational planner can enhance educational attainments of households beyond those achieved by a single national system.

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## 1. Introduction

How students should be allocated to schools to achieve educational goals is one of important debates on the construction of public school systems during the past few decades. Historically, many West European and Scandinavian countries transformed their secondary school systems from a selective to a comprehensive system during the 1960s and 1970s. Contemporaneously, some countries such as Germany and Singapore adopt a selective system based on entrance examinations to allocate students to secondary schools; other countries such as U.S., U.K. and South Korea employ a comprehensive system based on residential school districts to achieve the same goal.

Although a school system adopts either comprehensive, selective, or a combination of both methods at different levels of education (e.g., Japan) as a primary method of student allocation, there exists little agreement about the ultimate desirability of either system in education. Proponents of comprehensive systems argue that the major objective of a school system is to provide equal educational opportunities to all students irrespective of family and social backgrounds. They contend that by giving students rights to attend any school in the neighborhood, education can serve as a method for narrowing educational inequality between different income groups and correcting social segregation via intergenerational educational mobility (Jenkins et al., forthcoming; Leschinsky and Mayer, 1990; Oakes, 1985). They also believe that a comprehensive system performs better in terms of efficiency of educational production, because peer group effects in school benefit weak students but do not hurt strong students (Kang et al., 2007; Kerckhoff, 1986).

On the other hand, advocates of selective systems argue that a selective school maximizes educational outcomes because it is easier for teachers to instruct groups of students with a low variance of ability and for students to learn with similar-ability students in school (Fernandez and Gali, 1999; Gamoran and Mare, 1989; Slavin, 1990). They also believe that a selective system is more fair since student allocation is based upon ability alone, not upon family background. It is often the case under a comprehensive system that family incomes play an important role in student allocation to schools, because the levels of school and peer quality of a student are determined by the residential location and are reflected in house prices in the school district (Black, 1999; Gibbons and Machin, 2003). In addition, as usually observed in East Asian countries such as South Korea and Japan, the lack of school choice within the district gives rise to a large scale market for private tutoring (Kim and Lee, 2001). Private tutoring may function to transmit current inequality of family income to the next generation.

Disagreements about school systems do not remain at the theoretical level alone, but empirical studies also report conflicting findings about effects of school systems on efficiency and equity of education. Some studies (e.g., Bauer and Riphahn, 2006; Dustmann, 2004; Meghir and Palme, 2005; Pekkarinen et al., 2006; Shuetz et al., 2005) find that a selective (comprehensive) system reinforces (weakens) the impact of family background on educational outcomes, while others (e.g., Galindo-Rueda and Vignoles, 2005) report effects of school systems in the opposite direction. Studies also show mixed findings about inequality of educational outcomes under different school systems. Gorard and Smith (2004) and Hanushek and Woessmann (2006) find that a selective system exacerbates the inequality in educational outcomes. In contrast, Figlio and Page (2002) show ability tracking reduces the inequality of academic performance. Moreover, rare empirical studies on the impact of school systems on the level of educational outcomes also report inconclusive evidence regarding test scores and adulthood earnings (Hanushek and Woessmann, 2006; Kang et al., 2007).

Given such conflicting views and empirical findings on school systems, this paper attempts to shed light on how differently educational objectives are affected by different school designs and how sometimes conflicting empirical findings about the effects of school systems can be reconciled. First, we set up a simple economic model for a household's choice about consumption and educational investments for a child, and find the optimal decisions under comprehensive and selective school systems. Each school system is characterized by the role of a student's ability and the parent's choice of residential district and educational spending in the determination of quality of school

peers. We then compare the optimal choices and their impacts on efficiency and equity of educational outcomes across the two school systems. In the comparison, we employ as criteria educational expenditures, the role of family income and student ability in the determination of the educational outcome, and the variance and level of the outcome. Looking at differences in such variables across the two school systems, we examine what implications each school system has for efficiency and equity of education. Such a comparison will inform educational policymakers of the strengths and weaknesses of each school system in achieving educational goals.

Our theory of school systems gives rise to several points that fail to emerge in previous debates on school systems. Above all, it suggests that how a selective system is designed matters a great deal in a comparison between comprehensive and selective systems. Provided that a selective system is based on an entrance exam to assign students to schools, whether the exam signals a student's innate ability or ability affected by parents' educational spending (prior to the exam) yields different educational implications. For example, unlike the conventional belief, the effect of family income on educational outcomes can be weaker in a selective system than in a comprehensive system, if the former is based on an entrance exam biased toward innate ability (e.g., an IQ test). In addition, an entrance exam in a selective system may be designed in a way that it yields lower inequality in educational attainment than a comprehensive system offers. A judicious use of a selective system can at times achieve educational goals better than a comprehensive system does.

We also find that different households that are characterized by family income and the child's innate ability prefer different school systems; which household prefers which system depends on a design of the entrance exam in the selective system. Thus by offering multiple subsystems within a national school system (e.g., a comprehensive system in one region and a selective system in the other) and giving households a choice, the educational planner can enhance overall educational attainments beyond those that can be achieved by a single national system. We show this in our theory and in a simulation analysis.

The rest of the paper proceeds as follows: Section 2 discusses related literatures, and Section 3 introduces a basic model of a household's choice under student rationing systems---comprehensive and selective systems. In section 4, we discuss analytical results from the two systems in terms of efficiency and equity of education. Section 5 introduces a third school system---a market system where students are allocated through school markets. We compare outcomes of a market system with those of the rationing systems. In Section 6 we propose a school system with comprehensive and selective

allocations as subsystems, and examine its relative performance by means of a simulation. Section 7 concludes the paper.

## 2. Related Literature

While economic theories of school systems are relatively rare, the current paper is connected with the previous literature on economics of education in several ways. First, the most closely related are economic theories on peer effect and ability grouping in education. A selective schooling concerns ability grouping across schools; a comprehensive schooling concerns ability mixing across schools.<sup>1</sup> Benabou (1996) examines implications of ability grouping and mixing of students and workers for economic growth in the short-run and long-run. He argues that if the degree of complementarity of student abilities---negative externalities of weak students---in educational production is moderate, ability mixing tends to slow down the short-run growth but raise the long-run growth, since mixing homogenizes workers over time. However, even in the long run ability grouping may still be a better alternative, if complementarity in education is so strong that the cost of mixing in education exceeds the benefit of worker homogeneity. Lazear (2001) also investigates how educational outputs are affected by grouping methods of different-quality students in a classroom. He shows that the total educational output is maximized when students are grouped according to quality; yet he adds that ability mixing should be preferred if high-quality students can influence the output of low-quality students via peer interactions. Fernandez and Gali (1999) compare the relative performance of markets and tournaments---ability grouping by a test---in an economy with borrowing constraints. They show that tournaments dominate markets in terms of matching efficiency, and also in terms of aggregate consumption if tournaments are based on signaling technologies that are sufficiently responsive to ability variations relative to variations in expenditures.

Although they shed light on implications of different school systems and ability

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<sup>1</sup> In this paper our primary concern is about how to allocate students across schools, not across classes within a school given that the students are already placed to the school. For issues and impacts of within-school ability grouping and mixing on education, see, e.g., Argys et al. (1996), Betts and Shkolnik (2000), Figlio and Page (2002), Gamoran (1987), and Slavin (1990). In addition, up to section 4, we focus exclusively on public school systems, ignoring the presence of private schools. For issues concerning the role of ability grouping in the relationship between public and private schools, see, e.g., Epple et al. (2002).

grouping methods for efficiency and equity of education, such studies can be limited from the perspective of the construction of a school system. These studies generally draw educational outcomes of an alternative grouping method when a pool of students is exogenously given to be grouped across schools and classrooms. However, a pool of students to be grouped in a school may be subject to change when different grouping methods are adopted by different schools. For example, Epple et al. (2002) clearly show that sorting patterns of students between private and public schools vary according to whether public schools adopt ability tracking within school. There is a need for a theory of school systems to take parents' choices about grouping methods into account.

In addition, theoretical discussions of ability grouping in education generally put a primary focus on the efficiency aspect of educational production, and examine which method of student organization (i.e., ability grouping/tracking/streaming versus ability mixing/heterogeneous grouping) maximizes educational outcomes (e.g., Fernandez and Gali, 1999). In contrast, a rationale behind the introduction of a comprehensive schooling in Western Europe and several other countries has been to achieve an equal opportunity of education and cure social inequality mediated through education across generations (Leschinsky and Mayer, 1990). Illuminating the equity aspect of different school systems will balance and enhance our understanding about impacts of a school system on a society.

Second, there is believed to exist excessive private tutoring for primary and secondary school students in East Asian countries such as South Korea and Japan (Bray and Kwok, 2003). Parents and educational policymakers in these countries often call for a reform in the school system in a way that private tutoring is not required or at least reduced. The presence of excess private tutoring is at times explained by cultural factors such as a high educational motivation of parents in these countries (Bray, 1999, pp.69-70), which is difficult to manipulate by means of a school reform. It is possible, however, that unique systems of student allocation in these countries are responsible for the presence of such excess tutoring. For instance, if an entrance exam in a selective system is designed in a way that it is sufficiently responsive to variation in students' endowed ability relative to that in parents' educational spending (prior to the exam) for the children, the parents spend more on private tutoring under a comprehensive system than under a selective system.<sup>2</sup> Relying on an economic theory of school systems, we

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<sup>2</sup> A comparison of private tutoring patterns between South Korea and Japan is interesting in this respect, because both countries adopt a comprehensive system at the lower secondary level (i.e., middle school; grades 7 to 9), while South Korea employs a comprehensive system and Japan uses a selective system at

can understand better the relationship between a school system and the presence of excess private tutoring, and propose reform plans to reduce excessive tutoring.

Third, residential sorting is often discussed in connection with education (Epple and Romano, 2003; Fernandez, 2001, Nechyba, 2006). Such discussions are generally made in the context of the U.S. school system that is characterized by a comprehensive system. There is a lack of relevant economic theories that concern sorting patterns of households under alternative school systems. It is potentially important for a school design and its reforms to understand how households will be sorted under different systems of student allocation.

### 3. The model

Now we present our model. Consider the following Cobb-Douglas utility and educational production functions of a household that consists of a parent and one child;

$$u = x^\alpha A = x^\alpha (b\theta^\beta e^\gamma) \quad \text{where } \alpha + \beta + \gamma = 1, \quad 0 < \alpha, \beta, \gamma < 1 \quad (1)$$

where the household's total utility is given by the amount of private goods consumption ( $x$ ) and the child's educational outcome ( $A$ ). The educational outcome is determined by the child's time-invariant innate ability ( $b$ ); the average quality of peers in school ( $\theta$ ); and the amount of private education services ( $e$ ), which is provided by private educational institutions or private tutoring in the market.<sup>3</sup>

The current Cobb-Douglas form of utility and educational production functions plays an important role in drawing differences in educational implications across different school systems in this paper. The adoption of such a functional form is not unique to the current study. They are often employed by the economics of education literature (e.g., Epple et al., 2002; Epple and Romano, 1998; Ferreyra, forthcoming; Lazear, 2001; Nechyba, 2000). There is a trade-off between restrictiveness of the functions and the number of criteria that we can employ to assess different school

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the upper secondary level (i.e., high school; grades 10 to 12). In South Korea private tutoring is prevalent up to the upper secondary level; in Japan it is pronounced only up to the lower secondary level but relatively less widespread at the upper secondary level (Bray, 1999). In both countries, private tutoring is rare at the college level where student allocation is based on an exam-based selective system.

<sup>3</sup> To focus on choices on peers in school, we assume that every public school spends an equal amount of resources for a student.

systems. The more restrictive the functional forms are, the more dimensions (e.g., efficiency and equity) we can examine to evaluate the performance of school systems. For the current comparisons that look into several measures such as educational expenditures, the role of family income and student ability in the determination of the educational outcome, and the variance and level of the outcome, we believe that the Cobb-Douglas form of utility and educational production serves better, not worse at least, to highlight differences in educational implications across different systems.<sup>4</sup>

Now we consider two major school designs that are based on rationing: comprehensive and selective school systems. In a comprehensive system, parents choose a school district based on residential location, and their children are placed randomly into one school within the school district<sup>5</sup>; in a selective system, students take an entrance examination and are admitted to a school according to their ranking in the exam. (Later we consider a third system---a market system in which parents can purchase the quality of school peers at a given price.)

Under a comprehensive system, the quality of school peers ( $\theta$ ) is exogenously given to students once a school district is decided, hence parents can not choose the quality of peers in school directly. Within a district, students face the same average quality of peers in any school due to randomization. Nonetheless, parents can choose the quality of school peers indirectly by moving across school districts. The children's quality of a school district is reflected in house prices. Let us denote the children's average quality of a school district by  $d$  and its unit price by  $p_d$ . Parents spend a total of  $p_d d$  to purchase a house in a certain school district; a high house price in a

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<sup>4</sup> In order to check the robustness of our results to different functional forms of utility and educational production, we experiment with two alternative functions in Appendix: (1) CES utility with a Cobb-Douglas form of educational production; (2) Cobb-Douglas utility with a CES form of educational production. The first case produces the largely same results as those reported in the text; the second case, however, fails to produce analytical forms of key outcome variables, which makes it difficult to compare educational implications across different school systems.

<sup>5</sup> Under comprehensive systems, there are largely two methods of student placement within a school district. The first is to assign each student randomly to one school within the district. This method has been employed in South Korea since 1969 to allocate students to middle schools and general (or non-vocational) high schools within a school district. See OECD (1998, Chapters 1 and 2) for details. The second is to assign each student to one school within a district in consideration of the preferences of the student and parent. Such a method is more common in comprehensive systems than random assignment. In our analysis, however, we assume random assignment within a district for conceptual sharpness.

good school district is expressed by a high value of  $d$ , hence  $p_d d$ .

In the model, for simplicity, we assume that each school district has an identical living environment so that  $d$  is determined solely by the average quality of children residing in the district. We also suppose that there is no commuting cost of students between the house and school.

In contrast, under a selective system, the choice of a school district does not matter, because schools give admissions to students based on the entrance exam ranking alone, not on the school district. In this system parents choose the quality of school peers solely by raising the entrance exam ranking of the child. Under a selective system, the quality of school peers ( $\theta$ ) is determined by a child's innate ability ( $b$ ) and the parent's educational investments ( $f$ ) made prior to the exam to improve the ranking. Note that  $e$  denotes current educational investments that directly raise the current educational outcome ( $A$ ), while  $f$  concerns prior investments that directly improve the ranking of the entrance exam, but only indirectly related to the current outcome.  $f$  in a selective system corresponds to  $d$  in a comprehensive system in that the decision about them is made prior to attending school, although real time does not elapse.<sup>6</sup>

To characterize different systems of student allocation to schools, we introduce the following model for  $\theta$ :

$$\theta = b^{\delta_1} d^{\delta_2} f^{\delta_3} \quad (2)$$

A comprehensive system is characterized by  $\delta_2 = 1$  and  $\delta_1 = \delta_3 = 0$ , and a selective system by  $\delta_2 = 0$ ,  $\delta_1 > 0$  and  $\delta_3 \geq 0$ .  $\delta_2$  is equal to one in a comprehensive system that employs within-district randomization, because  $\theta$ , the average peer quality in a school, is ultimately determined by  $d$ , the average quality of children in a school district.<sup>7</sup> Instead of replacing it with one, we carry  $\delta_2$  in subsequent analysis for richness of interpretations. For a selective system we allow  $\delta_3$  to be equal to zero in

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<sup>6</sup> In a static model of the current paper, the introduction of educational investments prior to school entrance ( $f$ ) is somewhat unrealistic. Nevertheless, it highlights important aspects of differences in comprehensive and selective systems. Hur and Kang (2007) extend the current one-period model of school systems to a two-period model in which  $f$  is rendered unnecessary and the educational investments prior to the second-period school entrance are more realistically replaced by the educational investments made in the first period. For differences in modeling and other implications for school systems, see Hur and Kang (2007).

<sup>7</sup> If a within-district allocation of students is not random, then  $\delta_2$  may deviate from one.

order to invite an extreme design of the entrance exam. Not only can the educational planner choose between comprehensive and selective systems, but it can design a selective system by manipulating the relative levels of  $\delta_1$  and  $\delta_3$  in the entrance exam in a way that  $b$  matters more than  $f$  or vice versa. For example, an IQ test may be employed as the entrance exam, in which case  $\delta_3$  is close to zero and  $\delta_1$  remains positive.<sup>8</sup>

#### 4. A Household's Choice in Rationing Systems

Under rationing systems of student allocation, a parent faces the following budget constraint:

$$(1-t)y = x + p_d d + p_f f + p_e e, \quad (3)$$

where  $y$  is the income level of a household;  $t (\in (0,1))$  is an education tax rate charged to a household for an operation of public schools<sup>9</sup>;  $p_f$  is the price of  $f$ ; and  $p_e$  is the price of  $e$ . The price of  $x$  is assumed to be one for normalization. All the prices are exogenously determined in their own perfectly competitive markets.

Under rationing systems, the optimal quantity of each type of consumption, educational outcome and indirect utility function can be obtained as follows.

$$x^* = \left( \frac{\alpha}{\alpha + (\delta_2 + \delta_3)\beta + \gamma} \right) (1-t)y \quad (4)$$

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<sup>8</sup> Fernandez and Gali (1999) also consider different educational implications of different signaling technologies under a selective system. They show that if a signaling technology is sufficiently responsive to ability variation (i.e., relatively small  $\delta_3$  and large  $\delta_1$ ), a selective system based on tournaments delivers higher aggregate educational output and aggregate consumption than a market system. We find similar results below.

<sup>9</sup> A total of  $\sum t y$  is collected nationally as a total tax revenue and such revenue is divided equally to each public school by assumption. We employ such an assumption in order to focus exclusively on the impacts of student allocation across schools, setting monetary resource allocation across schools and districts aside. For issues related to school financing and the distribution of the resources, see, e.g., Fernandez and Rogerson (2003) and Hoxby (2001).

$$d^* = \left( \frac{\delta_2 \beta}{\alpha + (\delta_2 + \delta_3) \beta + \gamma} \right) \frac{(1-t)y}{p_d} \quad (5)$$

$$f^* = \left( \frac{\delta_3 \beta}{\alpha + (\delta_2 + \delta_3) \beta + \gamma} \right) \frac{(1-t)y}{p_f} \quad (6)$$

$$e^* = \left( \frac{\gamma}{\alpha + (\delta_2 + \delta_3) \beta + \gamma} \right) \frac{(1-t)y}{p_e}, \quad (7)$$

$$A^* = \left( \frac{\delta_2 \beta}{p_d} \right)^{\delta_2 \beta} \left( \frac{\delta_3 \beta}{p_f} \right)^{\delta_3 \beta} \left( \frac{\gamma}{p_e} \right)^\gamma \left[ \frac{(1-t)y}{\alpha + (\delta_2 + \delta_3) \beta + \gamma} \right]^{(\delta_2 + \delta_3) \beta + \gamma} \cdot b^{1 + \delta_1 \beta} \quad (8)$$

$$u^* = (x^*)^\alpha A^* \quad (9)$$

We below denote the outcomes of a comprehensive system by subscript  $c$  and those of a selective system by subscript  $s$ .

**Proposition 1: (Choice of Consumption and Educational Expenditures)** The optimal choices on consumption goods ( $x$ ), and expenditures on the quality of a school district ( $p_d d^*$ ), prior educational services ( $p_f f^*$ ) and current educational services ( $p_e e^*$ ) under the two systems are ranked as follows:

- (i) If  $\delta_2 > \delta_3$ , then  $x_c^* < x_s^*$ ,  $p_e e_c^* < p_e e_s^*$ ,  $p_d d_c^* > p_f f_s^*$  and

$$p_e e_c^* + p_d d_c^* > p_f f_s^* + p_e e_s^*.$$

- (ii) If  $\delta_2 \leq \delta_3$ , then  $x_c^* \geq x_s^*$ ,  $p_e e_c^* \geq p_e e_s^*$ ,  $p_d d_c^* \leq p_f f_s^*$  and

$$p_e e_c^* + p_d d_c^* \leq p_f f_s^* + p_e e_s^*.$$

**Proof:** Assign values of  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  that characterize comprehensive and selective systems, respectively. Then, the propositions are trivial from the comparison of (4), (5), (6) and (7). **QED**

Proposition 1 implies that the levels of a household's consumption and education-related spending vary between comprehensive and selective systems. Such levels do not

vary between the two systems alone, they also change within a selective system with values of  $\delta_3$  relative to  $\delta_2$ . If  $\delta_3$  is set less than  $\delta_2$ , for instance, the entrance exam in a selective system is relatively weakly responsive to variations in parents' spending prior to the exam. In such a case, households spend less on education and consume more in a selective system than in a comprehensive system. If  $\delta_3$  is set sufficiently large, however, the rankings are reversed: by investing in the entrance exam ( $f$ ) rather than purchasing a school district ( $d$ ), parents attempt to manipulate the quality of school peers of the child, spending more on education and consuming less in a selective system than in a comprehensive system.

**Proposition 2: (Elasticities of  $A$ )**

- (i) A household's elasticity of  $A$  with respect to  $b$  (i.e.,  $\frac{\partial \ln A}{\partial \ln b}$ ) is greater in a selective system than in a comprehensive system.
- (ii) A household's elasticity of  $A$  with respect to  $y$  (i.e.,  $\frac{\partial \ln A}{\partial \ln y}$ ) varies by different systems. If  $\delta_2 > \delta_3$ , then  $\frac{\partial \ln A_c^*}{\partial \ln y} > \frac{\partial \ln A_s^*}{\partial \ln y}$ ; if  $\delta_2 \leq \delta_3$ , then  $\frac{\partial \ln A_c^*}{\partial \ln y} \leq \frac{\partial \ln A_s^*}{\partial \ln y}$ .

**Proof:** From (8),  $\ln A^* = \text{constant} + (1 + \delta_1\beta)\ln b + [(\delta_2 + \delta_3)\beta + \gamma]\ln y$ . Then we can find the following;  $\frac{\partial \ln A^*}{\partial \ln b} = 1 + \delta_1\beta$  and  $\frac{\partial \ln A^*}{\partial \ln y} = (\delta_2 + \delta_3)\beta + \gamma$ . Assign values of  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  that characterize each system, respectively. Then, the propositions are trivial from the comparisons. **QED**

Like the conventional belief, a student's ability has a stronger influence on the educational outcome in a selective system than in a comprehensive system (see, e.g., Galindo-Rueda and Vignoles (2005) for empirical evidence). Unlike the conventional belief, however, the relative strength of the effect of family background on the educational outcome depends on the design of a selective system. If  $\delta_3$  is larger than  $\delta_2$ , i.e., the entrance exam in a selective system is strongly responsive to parents' spending before the exam, the effect of family income is greater in a selective system than in a comprehensive system. In such a case, rich parents in a selective system invest heavily to equip the children for the entrance exam. It strengthens the effect of family

income in a selective system relative to that in a comprehensive system. If  $\delta_3$  is less than  $\delta_2$ , however, the role of family income is smaller in a selective system than in a comprehensive system, because the entrance exam is strongly responsive to variations in students' endowed ability but only weakly responsive to parents' spending prior to the exam. Thus proposition 2 suggests that the educational planner can design the entrance exam in a way that it achieves a desired level of the relationship between a student's family background and educational outcome in a selective system. If such a design is difficult, if not impossible, in a selective system, a comprehensive system may offer an alternative level of such a relationship.

In line with the theoretical predictions, empirical studies find conflicting evidence on effects of family background on educational outcomes under different school systems. Our model suggests that it is due to different designs of student allocation under a selective system. Using international data sets, Shuetz et al. (2005) find that a selective (comprehensive) system reinforces (weakens) the impact of family background on educational outcomes. Similar findings are reported by Bauer and Riphahn (2006) for Switzerland, Dustmann (2004) for Germany, Meghir and Palme (2005) for Sweden and Pekkarinen et al. (2006) for Finland. In contrast, Galindo-Rueda and Vignoles (2005) show the role of parental background increased with the transition from a selective to a comprehensive system in the U.K. Waldinger (2006) and Brunello and Checchi (2006), however, find no evidence that family background has differential effects across comprehensive and selective systems, using international data sets.

**Proposition 3: (Inequality of the Educational Outcome: Variance of  $\ln A$ )**

- (i) Suppose that  $Cov(\ln b, \ln y) > 0$ , as is often empirically found. If  $\delta_2 \leq \delta_3$ , then the variance of  $\ln A$  is larger in a selective system than in a comprehensive system.
- (ii) If  $\delta_2 > \delta_3$ , then the relative size of the variance of  $\ln A$  is not determined uniformly between comprehensive and selective systems. It depends on parameter values,  $Var(\ln b)$ ,  $Var(\ln y)$  and  $Cov(\ln b, \ln y)$ .
- (iii) The variance of  $\ln A$  in a selective system incrementally increases as  $\delta_3$  rises. It is minimized when  $\delta_3$  is equal to zero.

**Proof:** From (8), we can obtain the variance of  $\ln A$  as follows.

$$Var(\ln A^*) = (1 + \delta_1 \beta)^2 Var(\ln b) + [(\delta_2 + \delta_3) \beta + \gamma]^2 Var(\ln y) + 2 \cdot (1 + \delta_1 \beta) [(\delta_2 + \delta_3) \beta + \gamma] \cdot Cov(\ln b, \ln y)$$

Assigning values of  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  that characterize each system, respectively, we have  $Var(\ln A_c^*) = Var(\ln b) + (\delta_2 \beta + \gamma)^2 Var(\ln y) + 2(\delta_2 \beta + \gamma) \cdot Cov(\ln b, \ln y)$  and

$Var(\ln A_s^*) = (1 + \delta_1\beta)^2 Var(\ln b) + (\delta_3\beta + \gamma)^2 Var(\ln y) + 2(1 + \delta_1\beta)(\delta_3\beta + \gamma) \cdot Cov(\ln b, \ln y)$ .  
 If  $\delta_2 \leq \delta_3$ ,  $Var(\ln A_c^*) < Var(\ln A_s^*)$ . If  $\delta_2 > \delta_3$ , however, either  $Var(\ln A_c^*) < Var(\ln A_s^*)$   
 or  $Var(\ln A_c^*) > Var(\ln A_s^*)$  mainly due to the presence of  $\delta_1$  in  $Var(\ln A_s^*)$ .

$$\frac{\partial Var(\ln A_s^*)}{\partial \delta_3} = 2\beta[(\delta_3\beta + \gamma)Var(\ln y) + (1 + \delta_1\beta)Cov(\ln b, \ln y)] > 0 \quad \text{and}$$

$$\frac{\partial^2 Var(\ln A_s^*)}{\partial \delta_3^2} = 2\beta^2 Var(\ln y) > 0 \quad \text{for all } \delta_3 \geq 0. \quad Var(\ln A_s^*) \text{ is minimized at } \delta_3 = 0.$$

**QED**

In contrast to the conventional belief, the inequality of the educational outcome, which is measured by the variance of  $\ln A$  in our model, is not necessarily larger in a selective system than in a comprehensive system. A comprehensive system allows the inequality of the educational outcome to depend on existing family income inequality (i.e.,  $Var(\ln y)$ ) and the correlation between family income and the child's ability (i.e.,  $Cov(\ln b, \ln y)$ ) without guard. In contrast, by setting  $\delta_3$  sufficiently low, a selective system can reduce contributions of both to the inequality of the educational outcome and achieve a variance of  $\ln A$  below the level offered by a comprehensive system. If  $\delta_3$  rises beyond a threshold, however, the inequality of the educational outcome can worsen rapidly, as opponents of selective systems warn, because  $Var(\ln A_s^*)$  incrementally increases over  $\delta_3$ .

Using the equations in the proof of proposition 3, we can draw the trajectory of  $Var(\ln A_s^*) - Var(\ln A_c^*)$  as  $\delta_3$  changes. It is shown in Figure 1.

<Insert Figure 1 here.>

In order to determine the value of  $Var(\ln A_s^*) - Var(\ln A_c^*)$  for a given  $\delta_3$ , the location of the intercept matters, because  $Var(\ln A_s^*) - Var(\ln A_c^*)$  is an incrementally increasing function of  $\delta_3$  for all  $\delta_3 \geq 0$ . The intercept is given by

$$\beta[\delta_1^2 \beta Var(\ln b) + 2\delta_1[Var(\ln b) + \gamma Cov(\ln b, \ln y)] - \delta_2[(\delta_2\beta + 2\gamma)Var(\ln y) + 2Cov(\ln b, \ln y)]]$$

. This is equal to or above zero if  $\delta_1 \geq \hat{\delta}_1$  and below zero if  $\delta_1 < \hat{\delta}_1$  where  $\hat{\delta}_1$  sets

the intercept equal to zero,  $\hat{\delta}_1 = (\sqrt{D} - Var(\ln b) - \gamma Cov(\ln b, \ln y)) / \beta Var(\ln b)$  and

$$D \equiv [Var(\ln b)]^2 + \gamma^2 [Cov(\ln b, \ln y)]^2 + 2(\delta_2 \beta + \gamma) Var(\ln b) Cov(\ln b, \ln y) + \delta_2 \beta (\delta_2 \beta + 2\gamma) \cdot Var(\ln b) \cdot Var(\ln y).$$

In summary, if  $\delta_3 \geq \delta_2$ , then  $Var(\ln A_s^*) > Var(\ln A_c^*)$ . If  $\delta_3 < \delta_2$ , however, different scenarios exist:  $Var(\ln A_s^*) \geq Var(\ln A_c^*)$  for all  $\delta_3 \geq 0$  if  $\delta_1 \geq \hat{\delta}_1$ . If

$\delta_1 < \hat{\delta}_1$ , on the other hand,  $Var(\ln A_s^*) \geq Var(\ln A_c^*)$  for  $\delta_3 \geq \hat{\delta}_3$  and

$Var(\ln A_s^*) < Var(\ln A_c^*)$  for  $\delta_3 < \hat{\delta}_3$ , where  $\hat{\delta}_3$ , the threshold, equates  $Var(\ln A_s^*)$

with  $Var(\ln A_c^*)$ .

Taken as a whole, proposition 3 shows that the ranking of inequality of the educational outcome between comprehensive and selective systems is not uniformly determined. It does not only depend on parameter values,  $Var(\ln b)$ ,  $Var(\ln y)$  and  $Cov(\ln b, \ln y)$ , but on the design of a selective system. Unlike the conventional belief, it is difficult, if not impossible, to predict the change in overall inequality of educational outcomes between comprehensive and selective systems, unless detail information is available. Provided that enough information can be obtained, however, the educational planner may design a selective system in a way that it yields lower inequality in educational outcomes than can be achieved by an alternative comprehensive system.

In line with the theoretical predictions, empirical studies find different directions of change in inequality of educational outcomes from an introduction of a new school system, probably due to different parameter values and selective designs,  $Var(\ln b)$ ,  $Var(\ln y)$  and  $Cov(\ln b, \ln y)$  of an education system under examination. Using international data sets, Gorard and Smith (2004) and Hanushek and Woessmann (2006) find that a selective system exacerbates the inequality in educational outcomes. In a study on ability tracking in the U.S., however, Figlio and Page (2002) show tracking helps low-ability students and reduces the inequality of academic performance.

**Proposition 4: (Level of the Educational Outcome: Size of  $\ln A$ )**

(i) A household's preference ordering of  $\ln A$  (i.e.,  $\ln(\frac{A_c^*}{A_s^*})$ ) is not determined

uniformly between comprehensive and selective systems. It depends on

parameter values and prices ( $p_d$  and  $p_f$ , but not  $p_e$ ).  $\ln(\frac{A_c^*}{A_s^*})$  increases (decreases) as  $p_f$  rises (falls) relative to  $p_d$ .<sup>10</sup>

(ii) A household's  $\ln(\frac{A_c^*}{A_s^*})$  is negatively related to the child's ability ( $b$ ). In contrast, the relationship between  $\ln(\frac{A_c^*}{A_s^*})$  and family income ( $y$ ) differs by the value of  $\delta_3$ . If  $\delta_2 > \delta_3$ , it is positively related to  $y$ ; if  $\delta_2 < \delta_3$ , it is negatively related to  $y$ ; if  $\delta_2 = \delta_3$ , it is independent of  $y$ .

**Proof:** From (8),  $\ln(\frac{A_c^*}{A_s^*}) = -\delta_1\beta \ln b + (\delta_2 - \delta_3)\beta[\ln y + \ln(1-t)] - (\delta_2\beta + \gamma)\ln(\alpha + \delta_2\beta + \gamma) + (\delta_3\beta + \gamma)\ln(\alpha + \delta_3\beta + \gamma) + \beta[\delta_2 \ln(\delta_2\beta) - \delta_3 \ln(\delta_3\beta) + \delta_3 \ln(p_f) - \delta_2 \ln(p_d)]$ . From the equation, the propositions can be obtained. **QED**

Proposition 4(i) suggests that it is difficult a priori to decide which system yields higher educational outcomes. This is in contrast to previous studies that usually support a selective system as an efficient method of student allocation to schools (e.g., Fernandez and Gali, 1999; Lazear, 2001). These studies ignore a possibility that parents may exercise choices about grouping methods per se and a pool of students to be grouped in school may be subject to change when different grouping methods are adopted by different schools. Our model addresses such a possibility via prices ( $p_d$  and  $p_f$ ). Because sorting of students into schools varies by the design, the ranking of the ultimate educational outcome mediated through peer effects is indeterminate between comprehensive and selective systems. We can only infer that given the design of a selective system (i.e., given  $\delta_1$  and  $\delta_3$ ), parents in general prefer a selective (comprehensive) system as  $p_d$  rises (falls) relative to  $p_f$ .

Rare empirical studies report inconclusive evidence on the impact of school systems on overall educational outcomes. While Kang et al. (2007) show that a transition from a selective to a comprehensive system in secondary education significantly increased the

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<sup>10</sup> Comparisons based on indirect utilities (i.e.,  $\ln(\frac{u_c^*}{u_s^*})$ ) do not change qualitative results.

average educational outcome (i.e., adulthood earnings) in South Korea, Hanushek and Woessmann (2006) find that school systems fail to yield significant differences in the average outcome (i.e., test score in secondary school), using international data sets.<sup>11</sup>

Proposition 4(ii) implies that households with different characteristics  $(b, y)$  prefer different systems, depending on the design of a selective system. To illustrate this, let us draw a line of  $(b, y)$  combinations that equates  $A_c^*$  with  $A_s^*$ , and identify which household prefers which system. Such a line is expressed by

$$y = C_1 \cdot b^{\delta_2/(\delta_2 - \delta_3)} \quad \text{where} \quad C_1 \equiv \frac{(\alpha + \delta_2\beta + \gamma)^{\delta_2 + \gamma/\beta} \left[ \frac{\delta_3\beta}{p_f} \right]^{\delta_3} \left[ \frac{p_d}{\delta_2\beta} \right]^{\delta_2}}{(\alpha + \delta_3\beta + \gamma)^{\delta_3 + \gamma/\beta}}$$
 and displayed in Figure 2.

<Insert Figure 2 here.>

Depending on values of  $\delta_3$ , there are three different types of the lines that equate  $A_c^*$  with  $A_s^*$ . If  $\delta_2 > \delta_3$ , the line is an upward-sloping curve; if  $\delta_2 = \delta_3$ , the line is a vertical straight line; if  $\delta_2 < \delta_3$ , the line is a downward-sloping curve. The preference ordering between comprehensive and selective systems varies according to the value of  $\delta_3$  relative to  $\delta_2$ .

In the first case where  $\delta_2 > \delta_3$ , given a child's ability, high-income (low-income) parents prefer a comprehensive (selective) system, because by purchasing a school district ( $d$ ) as opposed to investing in an entrance exam ( $f$ ), parents can more easily

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<sup>11</sup> Our theoretical model casts doubt on the capability of transnational comparisons to reveal the true strength of ability grouping (as opposed to ability mixing) in raising the level of educational outputs. Because each country possesses unique educational markets for school districts and private educational services, the prices are likely to differ across countries. Even if parents' preferences are identical, the educational output may vary across countries solely by the differences in the prices, not by the method of ability grouping in school. Hanushek and Woessmann (2006) attempt to overcome such a problem by applying difference-in-difference methods that use educational outputs of primary and secondary schooling. An implicit assumption of the paper is that there are no substantial differences in markets for school districts and private educational services between primary and secondary education in each country. A before-after comparison for a single country such as Kang et al. (2007) may also be a valid empirical strategy; a similar assumption is that there are no changes in the educational markets in the vicinity of the exogenous policy change.

manipulate the quality of school peers in a comprehensive system than in a selective system. Alternatively, given a family income, high-ability (low-ability) students prefer a selective (comprehensive) system, because a student's ability serves to enhance the quality of school peers in a selective system but does not in a comprehensive system.

In the second case where  $\delta_2 = \delta_3$ , given a family income, high-ability (low-ability) students prefer a selective (comprehensive) system for the same reason as above. In this case, a student's ability alone, but not family income, contributes to the preference ordering between the two systems. Since the elasticities of  $A$  with respect to  $d$  and  $f$  are the same, family income equally functions to improve the quality of school peers in comprehensive and selective systems.

In the third case where  $\delta_2 < \delta_3$ , it is same as in the previous two cases that given a family income, high-ability (low-ability) students prefer a selective (comprehensive) system. Unlike the previous cases, however, given a child's ability, high-income (low-income) parents prefer a selective (comprehensive) system, because income can be more effectively spent for investing in the entrance exam ( $f$ ) than for purchasing a school district ( $d$ ) due to a strong responsiveness of the exam to  $f$ .

In sum, proposition 4 suggests that the educational planner may design a selective system by means of the entrance exam in a way that parents value a selective system more or less than a comprehensive system. Depending on different designs of a selective system, each household can have different preference orderings for comprehensive and selective systems. Which design of a selective system is offered will ultimately depend on the objectives of education (e.g., efficiency and/or equity of education) that a society would like to achieve.

## 5. A Household's Choice in a Market System

As a third alternative of student allocation to schools, we consider a market system in which parents can purchase the quality of school peers ( $\theta$ ) at a given price ( $p_\theta$ ) charged by the perfectly competitive private school. Market systems are also analyzed by Epple and Romano (1998) and Fernandez and Gali (1999).

Under the market system, a parent maximizes the utility  $u = x^\alpha (b\theta^\beta e^\gamma)$  subject to a budget constraint  $y = x + p_\theta\theta + p_e e$ . There is no tax in this system ( $t = 0$ ), because only private schools exist.

The optimal quantity of each type of consumption and indirect utility function can be obtained as follows. With subscript  $m$  for a market system,

$$x_m^* = \alpha y \tag{10}$$

$$\theta_m^* = \beta \frac{y}{p_\theta}, \quad (11)$$

$$e_m^* = \gamma \frac{y}{p_e}, \quad (12)$$

$$A_m^* = \left( \frac{\beta}{p_\theta} \right)^\beta \left( \frac{\gamma}{p_e} \right)^\gamma b y^{\beta+\gamma}, \quad (13)$$

$$u_m^* = (x_m^*)^\alpha A_m^*, \quad (14)$$

Note, from equation (11), that if  $p_\theta$  is negatively related to a student's ability ( $b$ ) in the form of scholarships and fellowships, a school of a given quality is attended by students whose family income is high but ability is low, and those whose family income is low but ability is high. See Epple and Romano (1998) and Epple et al. (2002) for the same finding.

**Proposition 5: (Market versus Rationing Systems)**

$$(i) \quad \frac{\partial \ln A_m^*}{\partial \ln b} = \frac{\partial \ln A_c^*}{\partial \ln b} < \frac{\partial \ln A_s^*}{\partial \ln b}.$$

In addition,  $\frac{\partial \ln A_m^*}{\partial \ln y} = \frac{\partial \ln A_c^*}{\partial \ln y}$ . If  $\delta_3 \geq 1$ ,  $\frac{\partial \ln A_m^*}{\partial \ln y} \leq \frac{\partial \ln A_s^*}{\partial \ln y}$ ; if  $\delta_3 < 1$ ,

however,  $\frac{\partial \ln A_m^*}{\partial \ln y} > \frac{\partial \ln A_s^*}{\partial \ln y}$ .

(ii) Given that  $\delta_2 = 1$  in a comprehensive system,  $Var(\ln A_m^*) = Var(\ln A_c^*)$ .

(iii) If  $\delta_3 \geq 1$ ,  $Var(\ln A_m^*) < Var(\ln A_s^*)$ ; if  $\delta_3 < 1$ , however, either  $Var(\ln A_m^*) > Var(\ln A_s^*)$  or  $Var(\ln A_m^*) < Var(\ln A_s^*)$ . The inequality depends on parameter values,  $Var(\ln b)$ ,  $Var(\ln y)$  and  $Cov(\ln b, \ln y)$ .

(iv)  $\ln\left(\frac{A_c^*}{A_m^*}\right)$  is either positive or negative, depending on parameter values, a tax

rate ( $t$ ) and prices ( $p_d$  and  $p_\theta$ , but not  $p_e$ ).  $\ln\left(\frac{A_c^*}{A_m^*}\right)$  increases (decreases)

as  $p_\theta$  rises (falls) relative to  $p_d$ . Given that  $\delta_2 = 1$  in a comprehensive

system,  $\ln\left(\frac{A_c^*}{A_m^*}\right)$  is independent of  $b$  or  $y$ .<sup>12</sup>

- (v)  $\ln\left(\frac{A_s^*}{A_m^*}\right)$  is either positive or negative, depending on parameter values, a tax rate ( $t$ ) and prices ( $p_f$  and  $p_\theta$ , but not  $p_e$ ).  $\ln\left(\frac{A_s^*}{A_m^*}\right)$  increases (decreases) as  $p_\theta$  rises (falls) relative to  $p_f$ . It is positively related to  $b$ .  $\ln\left(\frac{A_s^*}{A_m^*}\right)$  is positively related to  $y$  if  $\delta_3 > 1$ ; negatively related to  $y$  if  $\delta_3 < 1$ ; and independent of  $y$  if  $\delta_3 = 1$ .
- (vi) Both  $\ln\left(\frac{A_c^*}{A_m^*}\right)$  and  $\ln\left(\frac{A_s^*}{A_m^*}\right)$  are negatively related with the tax rate  $t$ .

**Proof:** From equation (13),  $\ln A_m^* = \beta \ln\left(\frac{\beta}{p_\theta}\right) + \gamma \ln\left(\frac{\gamma}{p_e}\right) + \ln b + (\beta + \gamma) \ln y$  so that

we have  $\frac{\partial \ln A_m^*}{\partial \ln b} = 1$  and  $\frac{\partial \ln A_m^*}{\partial \ln y} = \beta + \gamma$ . A comparison with results in the proof of

proposition 2 reveals proposition 5(i); The variances of  $\ln A_m^*$ ,  $\ln A_c^*$  and  $\ln A_s^*$  are

$$\text{Var}(\ln A_m^*) = \text{Var}(\ln b) + (\beta + \gamma)^2 \text{Var}(\ln y) + 2(\beta + \gamma) \cdot \text{Cov}(\ln b, \ln y) \quad \text{and}$$

$$\text{Var}(\ln A_c^*) = \text{Var}(\ln b) + (\delta_2 \beta + \gamma)^2 \text{Var}(\ln y) + 2(\delta_2 \beta + \gamma) \cdot \text{Cov}(\ln b, \ln y) \quad \text{and}$$

$$\text{Var}(\ln A_s^*) = (1 + \delta_1 \beta)^2 \text{Var}(\ln b) + (\delta_3 \beta + \gamma)^2 \text{Var}(\ln y) + 2(1 + \delta_1 \beta)(\delta_3 \beta + \gamma) \cdot \text{Cov}(\ln b, \ln y).$$

Thus, if  $\delta_2 = 1$ ,  $\text{Var}(\ln A_c^*) = \text{Var}(\ln A_m^*)$ , which shows proposition 5(ii); From the preceding equations, if  $\delta_3 \geq 1$ ,  $\text{Var}(\ln A_s^*) > \text{Var}(\ln A_m^*)$ . If  $\delta_3 < 1$ , however,

$$\text{Var}(\ln A_s^*) - \text{Var}(\ln A_m^*) = \beta[\delta_3^2 \beta + 2\delta_3(\gamma \text{Var}(\ln y) + (1 + \delta_1 \beta) \text{Cov}(\ln b, \ln y))$$

$$+ \delta_1^2 \beta \text{Var}(\ln b) + 2\delta_1[\text{Var}(\ln b) + \gamma \text{Cov}(\ln b, \ln y)] - (\beta + 2\gamma) \text{Var}(\ln y) - 2\text{Cov}(\ln b, \ln y)].$$

Thus, either  $\text{Var}(\ln A_s^*) < \text{Var}(\ln A_m^*)$  or  $\text{Var}(\ln A_s^*) > \text{Var}(\ln A_m^*)$  mainly due to the presence of  $\delta_1$  in  $\text{Var}(\ln A_s^*)$ . This shows proposition 5(iii);

$$\ln \frac{A_c^*}{A_m^*} = (\delta_2 - 1)\beta \cdot \ln y + (\delta_2 \beta + \gamma) \ln(1 - t) + \beta[\delta_2 \ln(\delta_2 \beta) - \ln(\beta) + \ln(p_\theta) - \delta_2 \ln(p_d)]$$

$$- (\delta_2 \beta + \gamma) \ln(\alpha + \delta_2 \beta + \gamma) = (\beta + \gamma) \ln(1 - t) + \beta[\ln(p_\theta) - \ln(p_d)] \quad \text{given } \delta_2 = 1,$$

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<sup>12</sup> Comparisons based on indirect utilities (i.e.,  $\ln\left(\frac{u_c^*}{u_m^*}\right)$  and  $\ln\left(\frac{u_s^*}{u_m^*}\right)$ ) do not change qualitative results.

$$\text{and } \ln \frac{A_s^*}{A_m^*} = \delta_1 \beta \cdot \ln b + (\delta_3 - 1) \beta \cdot \ln y + (\delta_3 \beta + \gamma) \ln(1-t) \\ + \beta [\delta_3 \ln(\delta_3 \beta) - \ln(\beta) + \ln(p_\theta) - \delta_3 \ln(p_f)] - (\delta_3 \beta + \gamma) \ln(\alpha + \delta_3 \beta + \gamma).$$

Propositions 5(iv), (v) and (vi) are trivial. **QED**

Propositions 5(i) and 5(ii) suggest that once a market system is introduced, a comprehensive system is reduced to a market system as far as the elasticity of  $A$  with respect to  $b$  and  $y$ , and the variance of  $\ln A$  are concerned. If parents can purchase the child's peers in school ( $\theta$ ) directly at price  $p_\theta$ , they do not need to purchase  $d$  at the school district level and pay taxes. According to proposition 5(iv), however, the ranking of  $\ln A$  is not determined uniformly between comprehensive and market systems due to the presence of prices in the ranking.

Proposition 5(iii) indicates that it is difficult to predict the ranking of inequality in the educational outcome between selective and market systems. Using the equation in the proof, we can draw the trajectory of  $Var(\ln A_s^*) - Var(\ln A_m^*)$  over  $\delta_3$ , which is shown in Figure 3.

<Insert Figure 3 here.>

In order to determine the value of  $Var(\ln A_s^*) - Var(\ln A_m^*)$  for a given  $\delta_3$ , the location of the intercept matters, because  $Var(\ln A_s^*)$  is an incrementally increasing function of  $\delta_3$  for all  $\delta_3 \geq 0$ , as shown in proposition 3. The intercept is given by

$$\beta [\delta_1^2 \beta Var(\ln b) + 2\delta_1 [Var(\ln b) + \gamma Cov(\ln b, \ln y)] - (\beta + 2\gamma) Var(\ln y) - 2Cov(\ln b, \ln y)].$$

This is equal to or above zero if  $\delta_1 \geq \delta_1^*$  and below zero if  $\delta_1 < \delta_1^*$  where  $\delta_1^*$  sets the intercept equal to zero,  $\delta_1^* = (\sqrt{D'} - Var(\ln b) - \gamma Cov(\ln b, \ln y)) / \beta Var(\ln b)$  and

$$D' \equiv [Var(\ln b)]^2 + \gamma^2 [Cov(\ln b, \ln y)]^2 + 2(\beta + \gamma) Var(\ln b) Cov(\ln b, \ln y) \\ + \beta(\beta + 2\gamma) \cdot Var(\ln b) \cdot Var(\ln y).$$

In summary, if  $\delta_3 \geq 1$ ,  $Var(\ln A_s^*) > Var(\ln A_m^*)$ . If  $\delta_3 < 1$ , however, different scenarios exist:  $Var(\ln A_s^*) \geq Var(\ln A_m^*)$  for all  $\delta_3 \geq 0$  if  $\delta_1 \geq \delta_1^*$ ; if  $\delta_1 < \delta_1^*$ , on the

other hand,  $Var(\ln A_s^*) \geq Var(\ln A_m^*)$  for  $\delta_3 \geq \delta_3^*$  and  $Var(\ln A_s^*) < Var(\ln A_m^*)$  for  $\delta_3 < \delta_3^*$ , where  $\delta_3^*$  equates  $Var(\ln A_s^*)$  and  $Var(\ln A_m^*)$ . Thus, it is difficult, if not impossible, to predict the change in overall inequality of the educational outcome between selective and market systems, unless detail information is available.

Proposition 5(iv) implies that a household's preference ordering between comprehensive and market systems does not depend on its characteristics  $(b, y)$ . Every household likes either a comprehensive system or a market system. The ranking is determined solely by parameter values, a tax rate  $(t)$  and prices.

In contrast, the preference ordering between selective and market systems depends on a household's characteristics  $(b, y)$  according to proposition 5(v). Households with different characteristics  $(b, y)$  prefer different systems, depending on the design of a selective system. To illustrate this, let us draw a line of  $(b, y)$  combinations that equate  $A_s^*$  with  $A_m^*$ , and identify which household prefers which system in Figure 4. Such a

line is expressed by  $y = C_2 \cdot b^{\frac{\delta_3}{1-\delta_3}}$  where  $C_2 \equiv \left[ \frac{1-t}{\alpha + \delta_3\beta + \gamma} \right]^{\frac{\delta_3 + \gamma/\beta}{\delta_3}} \left[ \frac{\delta_3\beta}{p_f} \right]^{\delta_3} \frac{p_\theta}{\beta}$ .

<Insert Figure 4 here.>

As earlier, depending on the values of  $\delta_3$ , there are three different types of the lines that equate  $A_s^*$  with  $A_m^*$ . If  $\delta_3 < 1$ , the line is an upward-sloping curve; if  $\delta_3 = 1$ , the line is a vertical straight line; if  $\delta_3 > 1$ , the line is a downward-sloping curve.

If  $\delta_3 < 1$ , given a child's ability, high-income (low-income) parents prefer a market (selective) system, because by purchasing school peers  $(\theta)$  directly as opposed to investing in an entrance exam  $(f)$ , parents can more easily manipulate the quality of school peers in a market system than in a selective system. Alternatively, given a family income, high-ability (low-ability) students prefer a selective (market) system, because a student's ability serves to enhance the quality of school peers in a selective system but does not in a market system.

If  $\delta_3 = 1$ , a student's ability alone, but not family income, contribute to the preference ordering between market and selective systems, because family income equally functions to improve the quality of school peers both in market and selective

systems. Given a family income, high-ability (low-ability) students prefer a selective (market) system.

If  $\delta_3 > 1$ , it is same as in the previous two cases that given a family income, high-ability (low-ability) students prefer a selective (market) system. Unlike the previous cases, however, given a child's ability, high-income (low-income) parents prefer a selective (market) system, because income can be more effectively spent for investing in the entrance exam ( $f$ ) than for purchasing school peers ( $\theta$ ) due to a strong responsiveness of the exam to  $f$ . In sum, depending on different designs of a selective system, each household can have different preference orderings for selective and market systems. As in the comparison between comprehensive and selective systems, which design of a selective system is offered will depend on the objectives of education that a society would like to achieve.

Proposition 5(vi) indicates that parents in general do not like high tax rates. If a tax rate is lower than a threshold, however, they like rationing systems better than a market system. Thus, by making public schools more efficient and reducing the amount of taxes required for an operation of them, the educational planner can induce parents to prefer rationing systems to a market system.

**Proposition 6: (Educational Expenditures in Market and Rationing Systems)**

- (i) Given that  $\delta_2 = 1$  in a comprehensive system,  $E_m < E_c$  where  $E_j$  denotes total educational expenditures of a household in system  $j (= c, s, m)$ .
- (ii) If  $\delta_3 \geq 1$ , then  $E_m < E_s$ ; if  $\delta_3 < 1$ , however, there are two cases. When  $t \geq (1 - \delta_3)\beta$ ,  $E_m \leq E_s$ ; when  $t < (1 - \delta_3)\beta$ ,  $E_m > E_s$ .

**Proof:** From equations (5), (6), (7), (11) and (12), a household's total expenditures on

education are  $E_c \equiv p_d d_c^* + p_e e_c^* + ty = \left( \frac{t\alpha + \delta_2\beta + \gamma}{\alpha + \delta_2\beta + \gamma} \right) y$  in a comprehensive system,

$E_s \equiv p_f f_s^* + p_e e_s^* + ty = \left( \frac{t\alpha + \delta_3\beta + \gamma}{\alpha + \delta_3\beta + \gamma} \right) y$  in a selective system, and

$E_m \equiv p_\theta \theta_m^* + p_e e_m^* = (1 - \alpha)y$  in a market system.  $E_c - E_m = ty$  and

$E_s - E_m = \frac{\alpha y}{\alpha + \delta_3\beta + \gamma} [t + (\delta_3 - 1)\beta]$ . Given the equations, propositions (i) and (ii) are

trivial. **QED**

Since a comprehensive system is reduced to a market system once a market system is introduced, each household spends more exactly by the tax payment in a comprehensive system than in a market system. When a selective system is compared with a market system, however, total educational expenditures depend on the design of a selective system and a tax rate. With high tax rates above (below) a threshold, each household spends more (less) in a selective system than in a market system.

In combination with proposition 1, we infer that if  $\delta_3 \geq 1$ ,  $E_m < E_c \leq E_s$ ; if  $\delta_3 < 1$ ,  $E_c$  is the largest but the ranking between  $E_m$  and  $E_s$  depends on the tax rate: if  $t < (1 - \delta_3)\beta$ ,  $E_s < E_m < E_c$ , but if  $t \geq (1 - \delta_3)\beta$ ,  $E_m \leq E_s < E_c$ . An implication of such findings is that excessive educational spending as observed in East Asian countries is closely related to its comprehensive systems of student allocation. If excessive educational spending is an important problem of an educational system, then either a selective or market system may serve as an alternative to a comprehensive system.

## **6. A Policy Experiment: A School System with Double Subsystems**

An implication of the finding that households with different characteristics (i.e.,  $b$  and  $y$ ) prefer different systems is that by offering multiple subsystems within a school system and giving households a choice between them, the educational planner can enhance educational attainments as well as welfare of households beyond those that can be achieved by a single national system. For example, if a country adopts a national selective system in which students take an entrance exam and compete to enter a better school nationwide, our theory suggests that some households prefer such a system, while other households like an alternative comprehensive system; which household prefers which system depends on the design of the entrance exam in the selective system. In contrast to such a case, if the two subsystems are offered in two separate regions of a nation's education system, households with different sets of  $(b, y)$  sort (in consideration of moving costs) into the one that serves better to their needs, and the average educational attainment of students will rise.

Although double subsystems in a nation offer efficiency gains, how the equity of education that concerns the elasticities of  $A$  with respect to  $b$  and  $y$  and  $Var(\ln A)$  will be affected is not clear; the measures are nonlinearly (via weighted averages) dependent on parameters,  $Var(\ln b)$ ,  $Var(\ln y)$  and  $Cov(\ln b, \ln y)$  under the double subsystems. To evaluate consequences of the introduction of double

subsystems into a national education system, we employ a simulation method as follows: Given a distribution of  $(b, y)$ , we experiment with a system that has a comprehensive design in one region and a selective design in the other.<sup>13</sup> In this system, a student is subject to a comprehensive subsystem if she resides at the comprehensive region and to a selective subsystem if she resides at the selective region. A household must choose either region before the child enters a school in order to send her to the preferred school. Until relevant discussions are made later, we assume in the simulation that there is no cost of a household's moving between the two regions prior to the entrance to a school. We also suppose that there is no capacity constraint of schools: all students attend a school if they are qualified.

One thousand different sets of  $(b, y)$  are generated by drawing random numbers from a joint log-normal distribution given by

$$\begin{pmatrix} \ln b \\ \ln y \end{pmatrix} \sim Normal \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix} \right).$$

Following Epple and Romano (1998), we set  $Cov(\ln b, \ln y)$  equal to 0.4. We assign 0.3 to  $\beta$ , since peer effects literature reports the range of the effect of average peer quality on a student's achievement between 0.2 and 0.4 for elementary and secondary school students (Hanushek et al., 2003; Hoxby, 2000; Kang, 2007). To  $\gamma$  we allocate 0.2, because Card and Krueger (1996, p.37) summarize that a 10 percent increase in public school spending lead to about a 1-2 percent increase in subsequent earnings. Thus  $\alpha$  is equal to 0.5 in the simulation.<sup>14</sup> We set all of  $p_d$ ,  $p_f$  and  $p_e$  equal to one, as we have no useful information on them and wish to avoid arbitrary price differences.<sup>15</sup> We set  $t = 0.05$ .

Simulation results based on different combinations of  $\delta_1$  and  $\delta_3$  (given  $\delta_2 = 1$ )

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<sup>13</sup> We do not consider a market system in the simulation, because once a tax rate is given, comparisons are straightforward.

<sup>14</sup> Different values of  $\alpha$ ,  $\beta$  and  $\gamma$  in plausible ranges proposed in the text do not yield qualitative differences in simulation results.

<sup>15</sup> Proposition 4(i) indicates that different prices of  $p_d$  and  $p_f$  yield different rankings between  $A_c^*$  and  $A_s^*$ , while they do not affect other criteria of the comparison. Nevertheless, different prices do not change the finding that double subsystems outperform a single national system, since households will at any rate sort into the preferred subsystem.

are shown in Table 1. In the table,  $\Pr(A_c^* < A_s^*)$  denotes the proportion of students whose outcome will be larger in a national selective system than in a national comprehensive system;  $\frac{\partial \ln A}{\partial \ln b}$  and  $\frac{\partial \ln A}{\partial \ln y}$  are OLS estimates of  $\tau_1$  and  $\tau_2$ , respectively, of an econometric model ( $\ln A = \tau_0 + \tau_1 \ln b + \tau_2 \ln y + \varepsilon$ ) that uses the simulated data.  $Var(\ln A)$  and  $Mean(A)$  are calculated based on the simulated sample.

<Insert Table 1 here.>

As implied in propositions in section 3, both  $\frac{\partial \ln A_c^*}{\partial \ln b}$  and  $\frac{\partial \ln A_s^*}{\partial \ln b}$ ---each from a single national system---do not change over  $\delta_3$ .  $\frac{\partial \ln A_c^*}{\partial \ln y}$  is also constant over  $\delta_3$  but  $\frac{\partial \ln A_s^*}{\partial \ln y}$  rises with  $\delta_3$ .  $Var(\ln A_c^*)$  does not vary over  $\delta_1$  and  $\delta_3$ , but  $Var(\ln A_s^*)$  incrementally increases with  $\delta_3$ . While  $Var(\ln A_s^*) > Var(\ln A_c^*)$  for all  $\delta_3 > 0$  if  $\delta_1$  is either 0.75 or 1, the inequality reverses below a certain value of  $\delta_3$  if  $\delta_1$  is either 0.25 or 0.5.  $Mean(A_s^*)$  changes over  $\delta_1$  and  $\delta_3$  but  $Mean(A_c^*)$  is constant over both. The proportion of households that prefer a selective system to a comprehensive system falls with  $\delta_3$  for a given  $\delta_1$ .

If the two subsystems are offered in two separate regions of a nation, households freely (due to no moving cost) move between them to send the child to the preferred school. According to the first row of each panel of Table 1, the proportion of households that send the child to a selective system falls with  $\delta_3$  for a given  $\delta_1$ . In a system with double subsystems (denoted by subscript  $d$ ),  $\frac{\partial \ln A_d^*}{\partial \ln b}$  falls but  $\frac{\partial \ln A_d^*}{\partial \ln y}$  rises as  $\delta_3$  rises, because a comprehensive subsystem, which offers a lower level of  $\frac{\partial \ln A}{\partial \ln b}$  and a higher level of  $\frac{\partial \ln A}{\partial \ln y}$  than a selective subsystem, is increasingly attended by a higher proportion of students in the nation. For given values of  $\delta_1$  and  $\delta_3$ , the levels of both  $\frac{\partial \ln A_d^*}{\partial \ln b}$  and  $\frac{\partial \ln A_d^*}{\partial \ln y}$  are in the middle of the respective values of the comprehensive and selective designs, because the values of double subsystems are the weighted averages of those of single comprehensive and selective systems. For given values of  $\delta_1$  and  $\delta_3$ ,  $Var(\ln A_d^*)$  is also intermediate between  $Var(\ln A_c^*)$  and  $Var(\ln A_s^*)$ .

According to the simulation results, if  $\delta_1$  is either 0.25 or 0.5, then  $Var(\ln A_d^*)$  can be lower at certain levels of  $\delta_3$  than  $Var(\ln A_c^*)$ , which can be achieved by a national comprehensive system. For such values of  $\delta_1$  and  $\delta_3$ ,  $Var(\ln A_s^*)$  is smaller than  $Var(\ln A_c^*)$  because family income minimally functions to enhance a child's peer quality. As motivated earlier, in terms of the average educational outcome, the system with double subsystems outperforms the two single national systems, because households can choose the most efficient system for them.

If we introduce a fixed cost ( $T$ ) of moving between comprehensive and selective regions into the model, baseline results presented above largely hold. A major difference is that the moving costs prevent low-income households from moving to the preferred subsystem. In Figure 2 these households are represented by combinations of  $(b, y)$  below a dotted horizontal line at  $y = T$ . In addition, the slope of the line of  $(b, y)$  combinations that equate  $A_c^*$  with  $A_s^*$  changes in response to positive moving costs.

In the presence of moving costs, efficiency gains from double subsystems will be smaller than those in the case of no moving cost. Nevertheless, a system with double subsystems outperforms two single systems, unless  $T$  is exceptionally high. It also achieves an intermediate level of equity between single comprehensive and single selective systems.

## 7. Concluding Remarks

In this paper, we examine the impacts of different systems of student allocation on efficiency and equity of education. First, we argue that how a selective system is designed matters a great deal in a comparison between comprehensive and selective systems. If a selective system is based on an entrance exam to assign students to schools, whether the exam signals a student's innate ability or ability affected by parents' educational spending (prior to the exam) yields different educational implications. Unlike the conventional belief, the effect of family income on educational outcomes can be stronger in a comprehensive system than in a selective system, if the latter is based on an entrance exam biased toward innate ability. In addition, the educational planner may design an entrance exam in a selective system in a way that it yields lower inequality in educational attainment than a comprehensive system offers. A judicious use of a selective system can at times achieve educational goals better than a comprehensive system.

Second, the paper also finds that once introduced, a market system would dominate

a comprehensive system under certain conditions, and the former can be compared with a selective system. It is notable that a household's total educational expenditures are lowest under a market system whether an entrance exam under a selective system is biased toward either acquired or innate abilities, if the tax rate in rationing systems is sufficiently high. The ranking of the educational outcome, however, is not uniformly determined across the three systems.

Third, given that households with different characteristics (i.e.,  $b$  and  $y$ ) prefer different systems, a school system with double subsystems (e.g., comprehensive in one region and selective in the other) outperforms a single national system in terms of efficiency. It achieves an intermediate level of equity between single comprehensive and single selective systems.

Although useful policy implications can be drawn from our approach, it is not, of course, free of drawbacks. First, the model is based on partial equilibrium, where prices are fixed across different school systems. In the context of general equilibrium, however, the prices vary by the structure of markets. We do not model the changes in markets for  $d$ ,  $f$  and  $e$  in response to those in demand for each in different school systems. This would give us a new agenda for a further study.

Second, our education production is based on a Cobb-Douglas form, where peer-effects increase with a student's ability (i.e.,  $A = b\theta^\beta e^\gamma$ ). Such a form of peer effects is frequently used in economic theories of education (e.g., Fernandez and Gali, 1999; Lazear, 2001). Without concerns for prices, they usually find ability matching between a student and peers as an efficient method of student allocation to schools. As Lazear (2001) and Benabou (1996) point out, however, if high-ability students can more strongly affect low-ability students through interactions than vice versa, ability mixing in school and classroom yields more efficient outcomes. By assuming a Cobb-Douglas form, we avoid complications that may be introduced by different functional forms of educational production. Nevertheless, there is a need to employ different specifications of peer effects over ability in educational production for future research.

## Appendix: Two alternatives for the consumer's preferences

### (1) CES utility between $x$ and $A$

Consider a following utility function of a household that consists of a parent and one child;

$$u = [x^\rho + A^\rho]^{\frac{1}{\rho}} \quad \text{with} \quad A = b\theta^\beta e^\gamma \quad \text{and} \quad \theta = b^{\delta_1} d^{\delta_2} f^{\delta_3}, \quad (0 < \beta, \gamma < 1, \rho \leq 1).$$

The strategy for utility maximization is as follows. We choose the optimal  $x$  and  $A$  first and then choose the optimal  $\theta$  and  $e$  by maximizing the sub-function of  $A$ . The results are summarized as follows.

#### Rationing systems

$$x^* = \left( \frac{B}{P_A} \right) (1-t)y, \quad d^* = \left( \frac{\delta_2 \beta}{(\delta_2 + \delta_3)\beta + \gamma} \right) \frac{B(1-t)y}{p_d},$$

$$f^* = \left( \frac{\delta_3 \beta}{(\delta_2 + \delta_3)\beta + \gamma} \right) \frac{B(1-t)y}{p_f}, \quad e^* = \left( \frac{\gamma}{(\delta_2 + \delta_3)\beta + \gamma} \right) \frac{B(1-t)y}{p_e},$$

$$A^* = b \left( \frac{\delta_2 \beta}{p_d} \right)^{\delta_2 \beta} \left( \frac{\delta_3 \beta}{p_f} \right)^{\delta_3 \beta} \left( \frac{\gamma}{p_e} \right)^\gamma \left[ \frac{B(1-t)y}{(\delta_2 + \delta_3)\beta + \gamma} \right]^{(\delta_2 + \delta_3)\beta + \gamma} \cdot b^{1 + \delta_1 \beta}$$

$$u = [x^{*\rho} + A^{*\rho}]^{\frac{1}{\rho}}, \quad \text{where} \quad B = \frac{P_A}{\frac{1}{P_A^{\rho-1}} + P_A}$$

#### Market system

$$x_m^* = \left( \frac{B}{P_A} \right) y, \quad \theta_m^* = \left( \frac{\beta}{\beta + \gamma} \right) \frac{By}{p_\theta}, \quad e_m^* = \left( \frac{\gamma}{\beta + \gamma} \right) \frac{By}{p_e},$$

$$A_m^* = \left( \frac{\beta}{p_\theta} \right)^\beta \left( \frac{\gamma}{p_e} \right)^\gamma \left( \frac{B}{\beta + \gamma} \right)^{\beta + \gamma} b y^{\beta + \gamma}, \quad u_m^* = [(x_m^*)^\rho + (A_m^*)^\rho]^{\frac{1}{\rho}}, \quad \text{where} \quad B = \frac{P_A}{\frac{1}{P_A^{\rho-1}} + P_A}$$

The optimal choices are very similar to our original results in the text. One exception is the consumption on private goods. The expenditure on the private goods remains the same between comprehensive and selective systems.

### (2) CES function between $\theta$ and $e$

Consider a following utility function of a household that consists of a parent and one child;

$$u = x^\alpha A \quad \text{with} \quad A = b[\theta^\rho + e^\rho]^{\frac{1}{\rho}} \quad \text{and} \quad \theta = b^{\delta_1} d^{\delta_2} f^{\delta_3}, \quad (0 < \alpha, \beta, \gamma < 1, \rho \leq 1).$$

The strategy for utility maximization is as follows. We choose the optimal  $x$  and  $A$  first and then choose the optimal  $\theta$  and  $e$  by maximizing the sub-function of  $A$ . The results are summarized as follows.

### Rationing system - Comprehensive system

$$p_d d_c^* + p_e \left( \frac{p_d}{p_e} \frac{1}{\delta_2} \right)^{\frac{1}{1-\rho}} (d_c^*)^{\frac{1-\delta_2\rho}{1-\rho}} = \frac{(1-t)y}{\alpha+1}, \quad e_c^* = \frac{p_d}{p_e} \frac{1}{\delta_2} (d_c^*)^{\frac{1-\delta_2\rho}{1-\rho}},$$

$$A_c^* = b[(d_c^*)^{\delta_2\rho} + (e_c^*)^\rho]^{\frac{1}{\rho}}, \quad x_c^* = \left( \frac{\alpha}{\alpha+1} \right) (1-t)y$$

### Rationing system - Selective system

**Case 1:** If  $\delta_3 > 0$

$$p_f f_s^* + p_e \left( \frac{p_f}{p_e} \frac{1}{\delta_3} \frac{1}{b^{\delta_1\rho}} \right)^{\frac{1}{1-\rho}} (f_s^*)^{\frac{1-\delta_3\rho}{1-\rho}} = \frac{(1-t)y}{\alpha+1}, \quad e_s^* = \left( \frac{p_f}{p_e} \frac{1}{\delta_3} \frac{1}{b^{\delta_1\rho}} \right)^{\frac{1}{1-\rho}} (f_s^*)^{\frac{1-\delta_3\rho}{1-\rho}},$$

$$A_s^* = b[(b^{\delta_1} f_s^{*\delta_3})^\rho + (e_s^*)^\rho]^{\frac{1}{\rho}}, \quad x_s^* = \left( \frac{\alpha}{\alpha+1} \right) (1-t)y$$

**Case 2:** If  $\delta_3 = 0$

$$f_s^* = 0, \quad e_s^* = \frac{1}{\alpha+1} \frac{y(1-t)}{p_e}, \quad A_s^* = b[(b^{\delta_1})^\rho + (e_s^*)^\rho]^{\frac{1}{\rho}}, \quad x_s^* = \left( \frac{\alpha}{\alpha+1} \right) (1-t)y$$

### Market system

$$\theta_m^* = b^{\frac{\rho}{\rho-1}} \left( \frac{p_A}{p_\theta} \right)^{\frac{1}{1-\rho}} A_m^*, \quad e_m^* = b^{\frac{\rho}{\rho-1}} \left( \frac{p_A}{p_e} \right)^{\frac{1}{1-\rho}} A_m^*, \quad A_m^* = \left( \frac{1}{\alpha+1} \right) \frac{y}{p_A}, \quad x_m^* = \left( \frac{\alpha}{\alpha+1} \right) y$$

$$\text{where } p_A = \frac{1}{b} [(p_\theta)^{\frac{\rho}{\rho-1}} + (p_e)^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}}$$

Except in a special case of  $\delta_3 = 0$ , the functional form of the educational outcome ( $A$ ) fails to be determined. It would be difficult, if not impossible, to draw clear differences in educational implications across school systems, given the current functional forms of utility and educational production.

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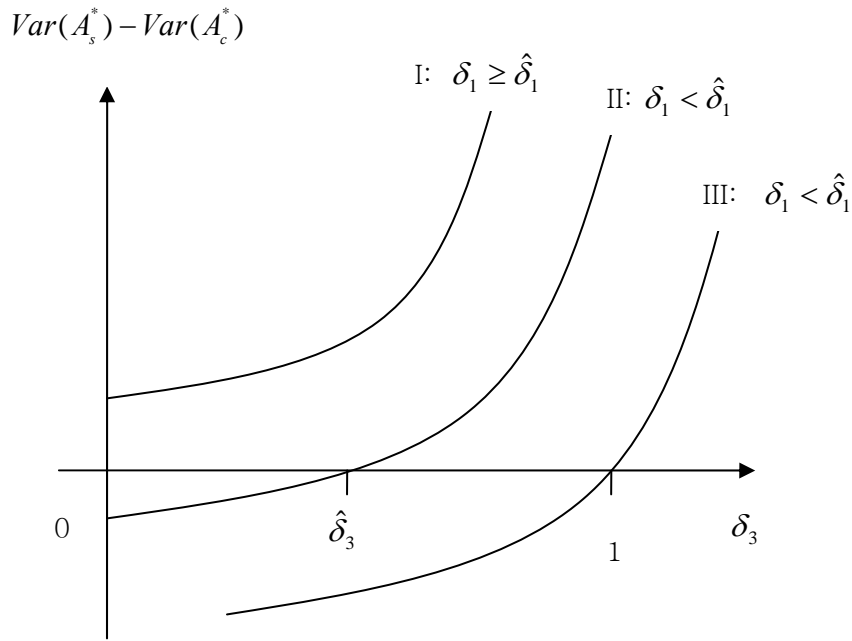


Figure 1: Trajectory of  $Var(\ln A_s^*) - Var(\ln A_c^*)$

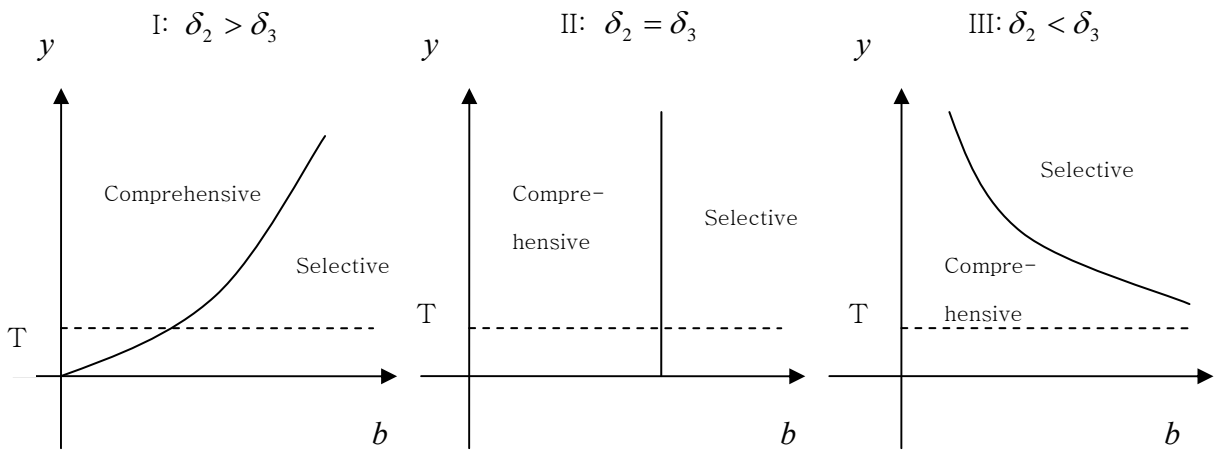


Figure 2: A line of  $(b, y)$  combinations that equate  $A_c^*$  with  $A_s^*$ ,

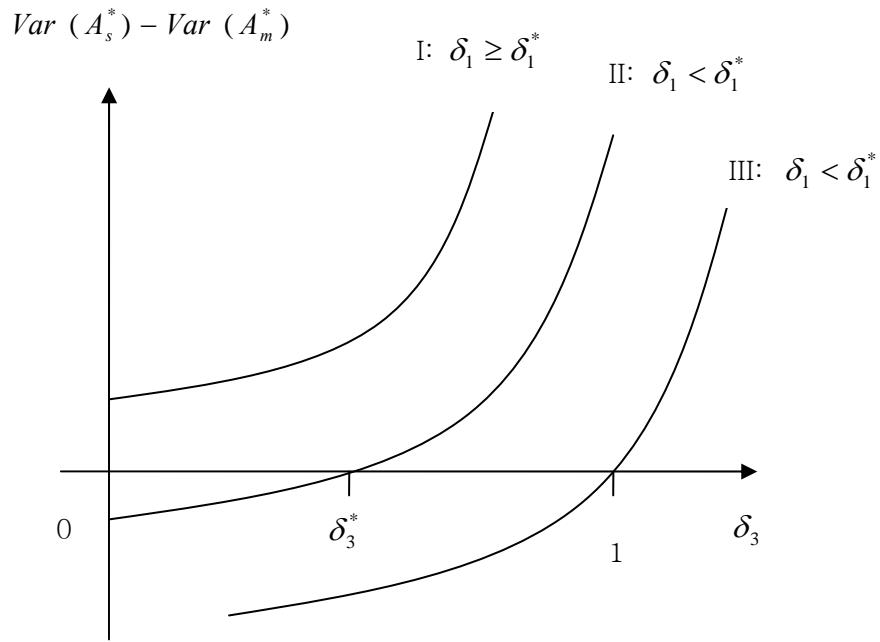


Figure 3: Trajectory of  $Var(\ln A_s^*) - Var(\ln A_m^*)$

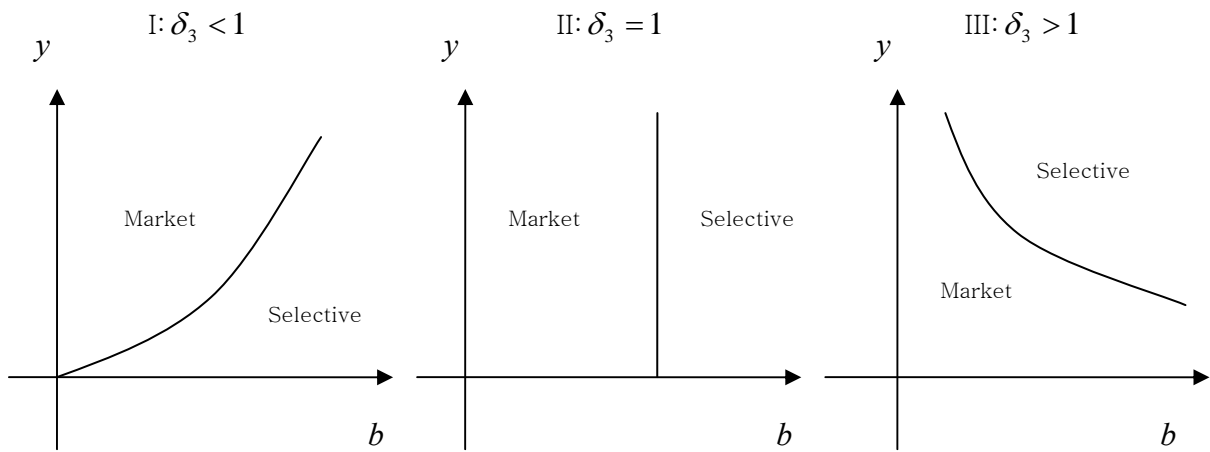


Figure 4: A line of  $(b, y)$  combinations that equate  $A_s^*$  with  $A_m^*$

Table 1: Simulation Results

		$\delta_1 = 0.25$								
		$\delta_3$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$\Pr(A_c^* < A_s^*)$			0.950	0.909	0.872	0.840	0.741	0.500	0.352	0.302
$\partial \ln A / \partial \ln b$										
Comprehensive			1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Selective			1.075	1.075	1.075	1.075	1.075	1.075	1.075	1.075
Double Subsystems			1.072	1.068	1.066	1.063	1.056	1.037	1.025	1.021
$\partial \ln A / \partial \ln y$										
Comprehensive			0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
Selective			0.200	0.260	0.320	0.380	0.440	0.500	0.560	0.620
Double Subsystems			0.217	0.285	0.344	0.400	0.455	0.500	0.521	0.537
$Var(\ln A)$										
Comprehensive			1.699	1.699	1.699	1.699	1.699	1.699	1.699	1.699
Selective			1.407	1.489	1.579	1.675	1.779	1.779	2.009	2.135
Double Subsystems			1.423	1.510	1.594	1.679	1.757	1.757	1.802	1.825
$Mean(A)$										
Comprehensive			1.083	1.083	1.083	1.083	1.083	1.083	1.083	1.083
Selective			1.484	1.297	1.226	1.193	1.181	1.184	1.201	1.229
Double Subsystems			1.514	1.334	1.256	1.209	1.186	1.189	1.213	1.252
		$\delta_1 = 0.5$								
		$\delta_3$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$\Pr(A_c^* < A_s^*)$			0.950	0.897	0.841	0.761	0.650	0.500	0.405	0.357
$\partial \ln A / \partial \ln b$										
Comprehensive			1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Selective			1.150	1.150	1.150	1.150	1.150	1.150	1.150	1.150
Double Subsystems			1.143	1.135	1.127	1.114	1.095	1.073	1.059	1.051
$\partial \ln A / \partial \ln y$										
Comprehensive			0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
Selective			0.200	0.260	0.320	0.380	0.440	0.500	0.560	0.620
Double Subsystems			0.216	0.286	0.348	0.408	0.462	0.500	0.524	0.542
$Var(\ln A)$										
Comprehensive			1.699	1.699	1.699	1.699	1.699	1.699	1.699	1.699
Selective			1.591	1.677	1.770	1.870	1.978	2.093	2.216	2.345
Double Subsystems			1.597	1.678	1.757	1.825	1.873	1.887	1.897	1.915
$Mean(A)$										
Comprehensive			1.083	1.083	1.083	1.083	1.083	1.083	1.083	1.083
Selective			1.619	1.417	1.342	1.307	1.296	1.302	1.322	1.354
Double Subsystems			1.636	1.437	1.356	1.316	1.302	1.310	1.336	1.377

Table 1: Simulation Results (Continued)

		$\delta_1 = 0.75$								
		$\delta_3$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$\Pr(A_c^* < A_s^*)$			0.941	0.863	0.786	0.713	0.592	0.500	0.436	0.398
$\partial \ln A / \partial \ln b$										
Comprehensive			1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Selective			1.225	1.225	1.225	1.225	1.225	1.225	1.225	1.225
Double Subsystems			1.212	1.195	1.177	1.155	1.131	1.110	1.094	1.084
$\partial \ln A / \partial \ln y$										
Comprehensive			0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
Selective			0.200	0.260	0.320	0.380	0.440	0.500	0.560	0.620
Double Subsystems			0.218	0.292	0.357	0.416	0.465	0.500	0.525	0.546
$Var(\ln A)$										
Comprehensive			1.699	1.699	1.699	1.699	1.699	1.699	1.699	1.699
Selective			1.787	1.876	1.973	2.077	2.189	2.307	2.434	2.567
Double Subsystems			1.779	1.848	1.906	1.952	1.979	1.987	1.996	2.013
$Mean(A)$										
Comprehensive			1.083	1.083	1.083	1.083	1.083	1.083	1.083	1.083
Selective			1.775	1.555	1.475	1.439	1.429	1.438	1.462	1.500
Double Subsystems			1.784	1.568	1.486	1.448	1.439	1.450	1.479	1.524
		$\delta_1 = 1.0$								
		$\delta_3$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$\Pr(A_c^* < A_s^*)$			0.913	0.821	0.751	0.665	0.564	0.500	0.446	0.408
$\partial \ln A / \partial \ln b$										
Comprehensive			1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Selective			1.300	1.300	1.300	1.300	1.300	1.300	1.300	1.300
Double Subsystems			1.275	1.246	1.219	1.193	1.167	1.146	1.130	1.119
$\partial \ln A / \partial \ln y$										
Comprehensive			0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
Selective			0.200	0.260	0.320	0.380	0.440	0.500	0.560	0.620
Double Subsystems			0.225	0.302	0.367	0.423	0.467	0.500	0.526	0.548
$Var(\ln A)$										
Comprehensive			1.699	1.699	1.699	1.699	1.699	1.699	1.699	1.699
Selective			1.994	2.087	2.188	2.295	2.411	2.533	2.663	2.800
Double Subsystems			1.964	2.006	2.042	2.072	2.084	2.091	2.100	2.116
$Mean(A)$										
Comprehensive			1.083	1.083	1.083	1.083	1.083	1.083	1.083	1.083
Selective			1.956	1.716	1.630	1.592	1.584	1.596	1.625	1.670
Double Subsystems			1.963	1.727	1.640	1.604	1.597	1.612	1.645	1.695